VDM-based Identification of Localized, Damage Induced Damping

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ABSTRACT

This paper presents and experimentally verifies a method for identification of structural damage. The work is focused on such damage types as cracks, delamination or excessive allowances, which may not cause significant stiffness degradation but induce noticeable additional damping. Damaged elements are located and the damage is assessed in terms of the damage-induced damping and also the stiffness degradation. The Virtual Distortion Method (VDM) is used for modeling of the modifications.

INTRODUCTION

This work is focused on identification of such damage patterns that can be poorly correlated with stiffness degradation levels of the damaged elements, but induce noticeable additional damping [1,2]. In such cases, it is not effective to base the identification process solely on monitoring of the related stiffness modifications. Other approaches have been proposed that focus on more sensitive features like the sub- and superharmonic resonances [3] or the adaptive likehood ratio [4]. However, the sensitivity of these features is strongly related to the level of damping present in the system and induced by the damage [4,5]. The methodology proposed here aims at detecting the damaged elements and assessing the damage scopes via identification of the damage-induced material damping, besides the stiffness degradation.

According to the excellent survey of damping identification methods by Srikantha Phani and Woodhouse [6], the method proposed here can be classified as a "matrix method" of damping identification. However, it seems to be more general, as it allows for a reduced-size local identification (full FRF matrix and response vectors of the damaged structure are then not required), a more flexible treatment of damping parameters of various origins, and unlike as in [7], for a simultaneous identification of modifications of stiffness. Moreover, if all the degrees of freedom (DOFs) related to the potentially damaged elements are measured, the (essentially

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nonlinear) identification problem can be effectively reduced into a simple linear problem.

The damping model assumed here is a generalized Rayleigh damping model which allows for separate modifications of damping coefficients in each element. The Virtual Distortion Method (VDM) [8] is used as a convenient methodology that allows for modeling of concurrent modifications of damping and stiffness at the element level. Moreover, as it is an essentially local approach, which models the structure in terms of so-called influence matrices, there is no need to formulate and solve the full equation of motion of the damaged structure. The structural response to an external excitation is composed of the response of the undamaged structure and of the residual, which is the responses of the intact system to a certain field of virtual distortions, which model the damage in chosen structural elements in terms of the related modifications of the material damping and stiffness. The effects of the distortions on the response are computed quickly via the VDM-specific influence matrix.

A preliminary experimental verification of the proposed method is performed using a cantilever beam with a single suspension element. The damage is modeled by including an additional damping element that modifies the suspension characteristics of the beam.

DAMPING MODEL

A generalized Rayleigh damping model is used. In the standard model, the damping matrix $\mathbf{C}$ is a weighted sum of the mass matrix $\mathbf{M}$ and the stiffness matrix $\mathbf{K}$, which are used to represent respectively the environmental and material damping factors, $\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K}$. The present work focuses on the material factors and generalizes the standard model to allow for independent modeling of material damping in separate structural elements. For notational simplicity it is assumed here that the considered structure is a truss; however, the approach can be straightforwardly applied also for other structures. In the case of a truss, the stiffness matrix can be directly related to the diagonal matrix $\mathbf{S}$ of element stiffnesses $E_i A_i$ by

$$\mathbf{K} = \mathbf{G}^{\mathrm{T}}\mathbf{L}\mathbf{S}\mathbf{G}, \tag{1}$$

where $\mathbf{L}$ is the diagonal matrix of element lengths l_i and $\mathbf{G}$ is the geometric (displacement-strain) matrix, which transforms the global displacements $\mathbf{u}$ to local element strains, that is $\varepsilon = \mathbf{G}\mathbf{u}$. Independent modifications of material damping at the element level is possible by expressing the damping matrix of the damaged structure in the following form:

$$\mathbf{C} = \mathbf{C}_0 + \mathbf{G}^{\mathrm{T}}\mathbf{L}\mathbf{S}\mathbf{C}_\beta\mathbf{G}, \tag{2}$$

where $\mathbf{C}_0$ represents the total damping of the original undamaged structure and $\mathbf{C}_\beta = \mathrm{diag}\{\beta_1, \ldots, \beta_{N_{\mathrm{el}}}\}$ is a diagonal matrix of the modifications to the material damping factors of the elements of the undamaged structure.

VDM-BASED REMODELING OF DAMPING AND STIFFNESS

Concurrent modifications of material damping and of stiffness are modeled at the element level by a single field of virtual distortions imposed on the original undamaged structure. The response of the damaged structure is thus modeled as the response of the unmodified structure distorted by the virtual distortions.

The analysis is performed in the frequency domain. A harmonic excitation $\mathbf{f}e^{\mathrm{i}\omega t}$ is considered, where $\mathbf{f}$ is a vector of the complex excitation amplitudes. The harmonic response of the original structure is denoted by $\mathbf{u}^{\mathrm{L}}$ (displacements) and ε^{L} (strains), which are assumed to be known. They can be found by solving the linear equation of motion of the undamaged structure, which yields the following two quasi-static formulations, which are equivalent by (1):

$$\begin{gathered}\left[-\omega^2\mathbf{M}+\mathrm{i}\omega\mathbf{C}_0+\mathbf{K}\right]\mathbf{u}^{\mathrm{L}}=\mathbf{f},\\ -\omega^2\mathbf{M}\mathbf{u}^{\mathrm{L}}+\mathrm{i}\omega\mathbf{C}_0\mathbf{u}^{\mathrm{L}}+\mathbf{G}^{\mathrm{T}}\mathbf{L}\mathbf{S}\varepsilon^{\mathrm{L}}=\mathbf{f}.\end{gathered} \tag{3}$$

Assume that some of the elements are damaged and that the damage is represented by two diagonal matrices: $\mathbf{C}_\beta$ of the modifications to the original material damping factors and $\Delta\mathbf{S}$ to the original matrix of element stiffnesses $\mathbf{S}$. The quasi-static equation of motion of the damaged structure is thus

$$-\omega^2\mathbf{M}\mathbf{u}+\mathrm{i}\omega\mathbf{C}_0\mathbf{u}+\mathrm{i}\omega\mathbf{G}^{\mathrm{T}}\mathbf{L}\mathbf{S}\mathbf{C}_\beta\varepsilon+\mathbf{G}^{\mathrm{T}}\mathbf{L}\left(\mathbf{S}+\Delta\mathbf{S}\right)\varepsilon=\mathbf{f}, \tag{4}$$

which could be directly solved to obtain the response of the damaged structure. However, it involves all DOFs and thus requires a solution of the full system, which is impractical in case of a large structure and localized damages. Moreover, the response is related to the modifications $\mathbf{C}_\beta$ and $\Delta\mathbf{S}$ in an essentially nonlinear way, and hence any direct solution of the corresponding full inverse problem would be numerically costly. The VDM models the damage locally (at the element level) by imposing response-coupled harmonic virtual distortions on the affected elements. Let ψ^0 denote the global vector of their complex amplitudes. Since they distort the unmodified structure, the equation of motion of the modeled structure is

$$-\omega^2\mathbf{M}\mathbf{u}+\mathrm{i}\omega\mathbf{C}_0\mathbf{u}+\mathbf{G}^{\mathrm{T}}\mathbf{L}\mathbf{S}\left[\varepsilon-\psi^0\right]=\mathbf{f}. \tag{5}$$

A comparison of (4) and (5) yields

$$\mathbf{A}\varepsilon=\psi^0, \qquad \text{or} \qquad \left(1-\mu_i-\mathrm{i}\omega\beta_i\right)\varepsilon_i=\psi_i^0. \tag{6}$$

In (6), $\mathbf{A}=\mathbf{I}-\boldsymbol{\mu}-\mathrm{i}\omega\mathbf{C}_\beta$, $\mathbf{I}$ is the identity matrix and $\boldsymbol{\mu}=\mathrm{diag}\left\{\mu_1,\mu_2,\ldots,\mu_N\right\}$, where $\mu_i=\hat{E}_i/E_i$ is the stiffness reduction ratio of the ith element. As the system matrix $\mathbf{A}$ is diagonal, each virtual distortion is proportional to the response of the involved element, and vanish if the element is non-damaged. Moreover, with respect to the response, it can be decomposed into the in-phase component, which models the stiffness modification, and the quadrature component, which models the damping modification. The original structure is linear and the response depends linearly on the virtual distortions,

$$\varepsilon = \varepsilon^{\mathrm{L}} + \mathbf{D}\psi^{0}, \tag{7a}$$

$$\mathbf{u} = \mathbf{u}^{\mathrm{L}} + \mathbf{B}\psi^{0}, \tag{7b}$$

where **D** and **B** are called the influence matrices and have to be pre-computed. Therefore, given the modification coefficients, the virtual distortions can be computed by

$$[\mathbf{I} - \mathbf{AD}]\,\psi^{0} = \mathbf{A}\varepsilon^{\mathrm{L}}, \tag{8}$$

which involves only the damaged elements and thus can be a much smaller system than (4). Given the distortions, the response of the damaged structure can be found by (7).

DAMAGE IDENTIFICATION

The damage identification problem, as defined in this work, is an inverse problem of identification of the modifications of material damping and stiffness at the element level, based on the response of the damaged structure (**u** and/or ε), which is measured in certain points. A straightforward solution would require a minimization of the residual of (4) with respect to all coefficients μ_i and β_i, which is a nonlinear problem that can be numerically costly, especially in case of a large structure and a localized damage. In this section, two alternative and significantly simpler approaches are proposed.

The first approach can be used if all the degrees of freedom (DOFs) that are related to the potentially damaged elements are measured, so that their (idealized) strain response ε is known. The identification problem can be then effectively reduced into a simple linear problem. First, the virtual distortions that result in the known response are computed by solving (7), which is reduced before to include only the measurement points and the potentially damaged elements. Notice that, in order to guarantee that it is uniquely solvable, enough many independent measurements have to be provided (with respect to the number of the potential damages). Then, given the distortions, the element-specific coefficients of damping and stiffness modifications can be computed directly by (6).

However, equation (6) can be used this way only if all the (idealized) strain responses ε_i of the potentially damaged elements are known. If they are not measured, the more general *second approach* has to be used, which uses (8) and (7) to compute the response and amounts to the minimization of the following objective function:

$$F(\boldsymbol{\mu},\boldsymbol{\beta}) = \frac{\|\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{M}}\|}{\|\boldsymbol{\varepsilon}^{\mathrm{M}}\|^{2}} + \frac{\|\mathbf{u} - \mathbf{u}^{\mathrm{M}}\|}{\|\mathbf{u}^{\mathrm{M}}\|^{2}} \tag{9}$$

which measures the discrepancy between the measured and the computed responses of the damaged structure.

EXPERIMENTAL VERIFICATION

A preliminary experimental verification of the proposed approach was performed using a 0.4 m long cantilever beam with a single suspension element, see Figure 1. The damage of the suspension element was simulated by mounting additionally a small damper with the identified damping coefficient. A numerical model of the undamaged structure was prepared and used to determine the influence matrices as well as the velocity responses to the external excitation and the harmonic virtual distortion.

The identification was performed in a range of frequencies from 72 Hz to 110 Hz. Since the strain of the damper was not measured, the second approach was used. Figure 2 compares the identified and the actual values of the coefficient β. The identification error depends on the frequency and ranges from 10% to 40%. The discrepancies can be caused by the crudeness of the numerical model of the undamaged structure used in computations, which will be improved. In further steps, more complex structures will be investigated, including a larger number of structural damages. Besides damping modifications, stiffness degradation will be also taken into account in further experimental work.

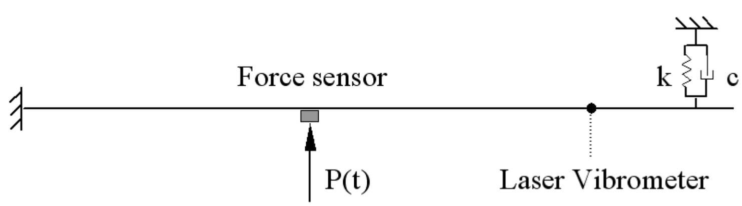

Figure 1. Experimental test stand: damaged structure

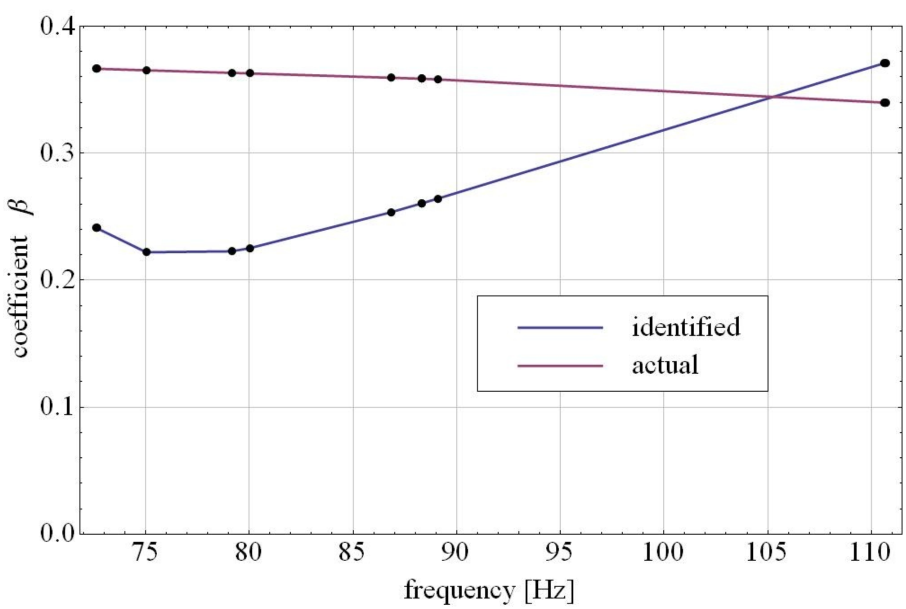

Figure 2. Identified and actual values of the damping coefficient β

CONCLUSIONS

This paper presents and experimentally verifies a method for modeling and identification of structural damage. It is focused on such damage types that may not cause significant stiffness degradation but induce noticeable additional damping. Concurrent modifications of damping and stiffness are modeled at the element level using the Virtual Distortion Method (VDM). The method is essentially local, so that there is no need to formulate and solve the full equation of motion of the damaged structure.

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